Corporate Risk - Scores & Rules

Exercise 1.

The number of rare losses of healthy firms observed in one year is distributed according to $\mathscr{B}(250, 0.01)$; the number of rare losses of poor health firms is three times larger in average.

1. What is the chance of five rare losses in one year for (i) healthy firms (ii) poor health firms?

One observes five rare losses during last year for J&B.

2. Would you say J&B is healthy, given there are twice as many healthy firms than poor health firms?

Exercise 2.

Daily profits of healthy companies (70% of all companies) are distributed according to $\mathscr{N}(10, 9)$. Daily profits of poor health companies are Gaussian, but the mean profit is 20% smaller and the standard deviation is 50% larger.

1. Compare the probability to earn at least 13 in each class of company.
2. John's company just earned more than 13; what about the class of the company?

Exercise 3.

Annual losses are distributed according to $\mathscr{N}(3, 1)$ for healthy firms, or to $\mathscr{N}(5, 1)$ for firms close to bankruptcy. Healthy firms are four times as numerous as non healthy firms.

1. Compute Type I and Type II errors.
2. What is the chance of being wrong when estimating one firm status, given the annual loss?

Exercise 4.

30% of businesses are risky, the other ones are safe. Assuming one riksy business to be safe (type II error) has 40% chance and costs 2. Assuming one safe business to be risky (type I error) has 10% chance and costs 1. What is the mean cost of classification?

Exercise 5.

Table 1 indicates the proportions of healthy/frail firms having high/medium/low degree of ethical values.

	ethical values		
	plus	neutral	minus
healthy	40%	30%	30%
frail	20%	20%	60%

Table 1

Healthy firms are 40% of all firms and Birdy Bank invests in firms with high ethical values.

1. What is the chance that Birdy invests in a frail firm?
2. What proportion of firms are frail among Birdy's investments?
3. What is the chance of investing either in a frail firm with high ethical values or in a healthy firm with medium/low ethical values?

Exercise 6.

Table 2 indicates the liquidity of ten companies according to the status: safe/frail.

safe	$8, 7, 6, 8, 5, 6$
frail	$2, 5, 4, 4$

Table 2

1. Display the data of Table 2 on one axis.

	liquidity		
	weight	mean	variance
safe	$w_s =$	$m_s =$	$v_s =$
frail	$w_f =$	$m_f =$	$v_f =$

Table 3

2. Compute the conditionnal statistics of Table 3.

Let us consider one company with liquidity x but unknown status. In order to estimate the status, John proposes to compute the score: $s_1(x) = x - (m_s + m_f)/2$ and to consider as safe any company with a positive score: $s_1(x) > 0$. Sam proposes to replace John's score with: $s_2(x) = (x - m_f)^2/v_f - (x - m_s)^2/v_s - 2\log(w_f/w_s) - \log(v_s/v_f)$.

3. What is the estimated class for a company the liquidity of which is 5.1?

4. Does the same value of the liquidity leads to undecision in both cases?

5. Discuss the advantages of the two scores and classification rules.

Exercise 7.

A population of firms is described by two financial ratios: R_1, R_2 and divided into healthy and frail firms. Table 4 indicates the statistical features of each ratio in each class of firm. The covariance between R_1 and R_2 equals 2 for safe firms and the covariance equals 3 for frail firms. The random vector (R_1, R_2) is assumed to be Gaussian in each class of firm.

		R_1		R_2	
	weight	mean	variance	mean	variance
safe	$w_s = 0.8$	$m_{1,s} = 8$	$v_{1,s} = 4$	$m_{2,s} = 10$	$v_{2,s} = 3$
frail	$w_f = 0.2$	$m_{1,f} = 5$	$v_{1,f} = 5$	$m_{2,f} = 7$	$v_{2,f} = 4$

Table 4

1. What is the chance for a firm with: $(R_1, R_2) = (9, 6)$ to be safe?

2. Propose two scores and rules enabling to allocate any firm with known ratio to one class.

Exercise 8.

Table 5 indicates the distribution of ordinal grades given by Mowglie's and Flesh Ratings to companies within two classes of risk: $+/-$.

		Flesh Ratings			*Mowglie's*	
		A	B	C	A	B
risk	$+$	20%	30%	50%	20%	80%
	$-$	60%	30%	10%	90%	10%

Table 5

Class '+' is 20% of all firms. Mowglie's and Flesh Ratings' grades are independent.

1. Define a score enabling to affect any company to one class, given Mowglie's and Flesh Ratings grades.

2. What is the estimated class of AT&T that obtains A from Flesh Ratings, B from Mowglie's?

Assigning a firm to class '+' wheareas it belongs to class '−' costs 20. Assigning to class '−' a firm that belongs to class '+' costs 10.

3. What is the average cost of misclassification?

Exercise 9.

Table 6 indicates the grades given by Flesh Ratings and Moglie's to ten companies.

We consider Table 5 as the true conditionnal distributions of the grades. Any company quoted $x \in \{\text{A,B,C}\}$ by Flesh Ratings and $y \in \{A, B\}$ by Mowglie's is denoted (x, y). Let us define:

- the score $t^+(x, y)$ as the chance for (x, y) to belong to Class +

- the classification rule $\mathscr{R}_\tau$ that allocates (x, y) to Class + if and only if $t^+(x, y) > \tau$.

company	1	2	3	4	5	6	7	8	9	10
Flesh Ratings	A	A	B	B	B	C	B	B	C	C
Mowglie's	A	A	A	A	B	B	B	A	B	A

Table 6

1. Let us consider the classification rule $\mathscr{R}_{0,5}$.
   (a) Estimate the class of each firm of Table 6.
   (b) Provide the confusion table: true class × estimated class.
   (c) What is the number of (i) false positive (ii) false negative (iii) true positive (iv) true negative?
   (d) Compute the false positive rate (FPR) and true positive rate (TPR).
2. Compute (FPR_τ, TPR_τ) for $\tau \in \{0; 0,4; 0,5; 0,6; 1\}$.
3. Plot the ROC curve of the classifier and compute AUC.

Exercise 10.

Table 7 indicates the grades given by Flesh Ratings to one sample of companies as the EBIT of each company.

company	1	2	3	4	5	6	7	8	9	10
Flesh Ratings	A	A	B	B	B	A	B	B	B	B
EBIT	6.1	8.3	7.1	4.1	4.2	6.2	3.1	2.1	4.7	1.9

Table 7

Let us assume:
- two classes of risk: $+/-$
- EBIT is distributed according to $\mathscr{N}(4,2)$ in Class '$-$', according to $\mathscr{N}(6,3)$ in Class '$+$'
- 80% of A in Class '$+$', 30% of A in Class '$-$'
- EBIT and Flesh Ratings grade are independent.

$x \in \{\text{A,B}\}$ denotes the quote given by Flesh Ratings and $y \in \mathbb{R}$ the value of EBIT. Let us define:
- the score $t^+(x,y)$ as the chance for (x,y) to belong to Class '$+$'
- the classification rule $\mathscr{R}_\tau$ that allocates (x,y) to Class '$+$' if and only if $t^+(x,y) > \tau$.

1. Let us consider the classification rule $\mathscr{R}_{0,5}$.
   (a) Estimate the class of each firm of Table 6.
   (b) Provide the confusion table: true class × estimated class.
   (c) What is the number of (i) false positive (ii) false negative (iii) true positive (iv) true negative?
   (d) Compute the false positive rate (FPR) and true positive rate (TPR).
2. Compute (FPR_τ, TPR_τ) for $\tau \in \{0; 0,4; 0,5; 0,6; 1\}$.
3. Plot the ROC curve of the classifier and compute AUC.